Conductor Loss in Hollow Waveguides Using a Surface Integral Formulation

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Abstract—**The power-loss method along with a surface integral formulation has been used to compute the attenuation constant in hollow waveguides of arbitrary cross-section. An E-field integral equation is developed for the surface electric currents which is transformed into a matrix equation using the method of moments. An iterative technique, i.e., Muller's method is used to obtain the relation between the propagation constant and frequency. The attenuation constants have been calculated and formulated for various waveguides and are in good agreement with published data.**

I. Introduction

NUMEROUS papers are available in the literature for the analysis of waves propagating in hollow waveguides of arbitrary cross-section [1]–[4]. Some of the papers in this area are the works done by Swaminathan *et al.* [1], Spielman and Harrington [2], Bristol [3] and Kim *et al.* [4]. These papers however deal with hollow waveguides made up of perfectly conducting walls supporting waves at low frequencies. The work presented here is an extension of [1] and deals with the computation of the attenuation constant of hollow waveguides supporting waves at high frequencies.

At millimeter wave frequencies, the finite conductivity of the waveguide walls in hollow waveguides produces an attenuation in the wave propagating in the waveguide. To accurately characterize the hollow waveguide at millimeter wave frequencies, an estimate for the attenuation constant is necessary. Since the finite conductivity of the waveguide walls produces this attenuation, the conductivity of the waveguide walls has to be taken into consideration while calculating the fields produced by the wave propagating in the waveguide. As long as this conductor loss is small, the power-loss method can be used to compute the attenuation constant [5].

Using the surface equivalence principle the waveguide walls are replaced by equivalent electric surface currents radiating into free space. Enforcing the appropriate boundary condition an E-field integral equation (EFIE) is developed for these currents. Method of moments ([6]) with pulse expansion and point matching testing procedure is used to transform the integral equation into a matrix one. The next step in the calculation of the attenuation constant is to obtain a relationship between the propagation constant and frequency. For this purpose the matrix equation is rearranged into a diferent form which contains the minimum eigenvalue of the moment matrix.

An iterative technique, i.e., Muller's method [7] is used to find the frequency at which the minimum eigenvalue goes to zero. The main advantages of this technique are that it converges quadratically in the vicinity of a root, does not require the evaluation of any derivatives, and searches for complex roots even when those roots are not simple.

Once the relationship between the propagation constant and frequency is known, the fields inside and on the surface of the waveguide are calculated using the eigenvector pertaining to the minimum eigenvalue of the moment matrix. This is necessary to compute the attenuation constant. Normalized values of the attenuation constants have been calculated and formulated for various waveguides. A comparison has been made for a rectangular waveguide and it has been found that our results are in very good agreement with published data.

The power-loss method coupled with the surface integral formulation [1], [8] is used in this paper to analyze hollow waveguides of arbitrary cross-section.

II. Theory

The power-loss method which has been used in this paper for calculating the attenuation constant assumes that the losses are low at high frequencies. Hence it can be safely assumed that the finite conductivity of the waveguide walls has only a small effect on the field configuration within the waveguide. Due to the large conductivity of the waveguide walls, the magnetic field tangential to the wall depends only slightly on the wall conductivity. Thus the tangential magnetic field strength computed for perfectly conducting walls remains the same when the walls are assumed to have finite conductivity.

Based on the power-loss method [5], the attenuation constant is defined as

$$\alpha = \frac{P_L}{2P_T} \tag{1}$$

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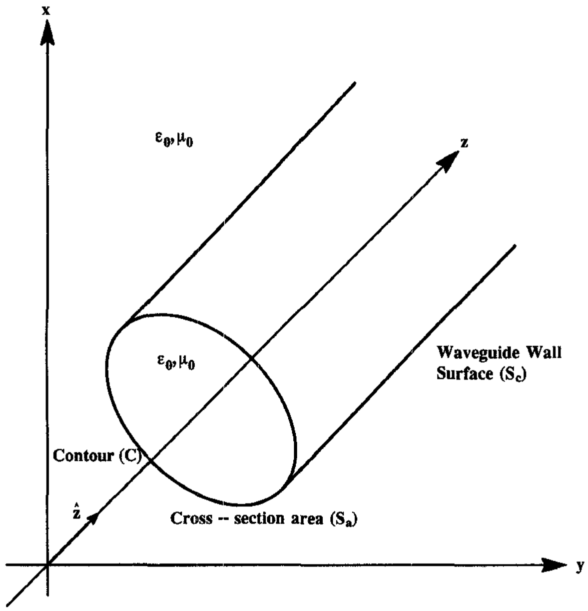

Fig. 1. Geometry of hollow waveguide.

where

$$P_L = \frac{1}{2} R_s \oint_c |H_{\tan}|^2 \, dl$$

$$P_T = \frac{1}{2} \iint_{S_a} \mathrm{Re}\,(\overline{E} \times \overline{H}^*) \cdot \overline{z} \, ds.$$

In the above equations, P_L is the power lost per unit length, P_T is the power transmitted, $H_{\tan}$ is the magnetic field tangential to the waveguide walls assuming that the walls are perfectly conducting, R_s is the surface resistance of the guide walls, $\overline{E}$ is the electric field inside the waveguide and $\overline{H}^*$ is the complex conjugate of the magnetic field existing inside the waveguide. It is further assumed that the wave propagates along the z-direction inside the waveguide. In (1) $\overline{z}$ is the z-directed unit vector. As is obvious from the above equations P_L is given by a contour integral along the contour (C) making up the cross-section of the waveguide and P_T by a surface integral on the cross-sectional surface area (S_a) of the waveguide (Fig. 1). The surface resistance R_s at any angular frequency ω is given bv

$$R_s = \sqrt{\frac{\omega \mu_o}{2\sigma}} \tag{2}$$

where μ_o is the free space permeability and σ is the conductivity of the waveguide walls. Equation (1) represents the formula for computing the attenuation constant of a hollow waveguide with finite cross-section and infinite along the direction of propagation.

Due to attenuation of the wave travelling in the hollow waveguide, the complex propagation constant is given by

$$\gamma = \alpha + j\beta \tag{3}$$

In (3), β is the propagation constant for a waveguide with perfectly conducting walls.

The attenuation α can be computed from (1) by calculating P_L and P_T. This can be a very cumbersome process for waveguides with arbitrary cross-sections due to the absence of any analytical expressions for the fields inside the waveguide. This paper uses a surface integral technique to calculate the fields existing in the waveguide made up of walls with finite conductivity.

III. Surface Integral Formulation

Consider a hollow conducting waveguide with arbitrary cross-section and with infinite extension in the z-direction (Fig. 1) which is the direction of propagation of the electromagnetic wave. The waveguide is completely filled with homogeneous dielectric (air in the hollow waveguide case) with permeability μ_o and permittivity ϵ_o. Since the waveguide does not radiate into the surrounding medium due to the presence of the perfectly conducting walls ($\sigma \to \infty$) the electric and magnetic fields at any point external to the waveguide are zero.

Using the surface equivalence principle [9, ch. 3] the original problem can be reduced to an equivalent one as shown in Fig. 2.

Fig. 2(a) shows the original problem, where S_c denotes the surface of a perfectly conducting cylinder which represents the hollow waveguide. The space surrounding the cylinder is assumed to be free space and characterized by the parameters (ϵ_o, μ_o). The surface S_c has unit normal vector $\overline{n}$ and unit tangent vector $\overline{t}$ which satisfy the following equation:

$$\overline{n} \times \overline{t} = \overline{z} \tag{4}$$

Fig. 2(b) shows the equivalent problem. The perfectly conducting cylinder in Fig. 2(a) is now replaced by a surface electric current $\boldsymbol{J}$ residing on S_c. According to the surface equivalence principle it is postulated that this current produces the exact fields ($\boldsymbol{E}_{\text{int}}$, $\boldsymbol{H}_{\text{int}}$) inside the cylinder and zero fields outside the cylinder. The magnitude of this surface electric current is given by the discontinuity of the tangential magnetic field on the surface of the cylinder. Since the magnetic field outside the cylinder is zero so,

$$\overline{J} = \overline{n} \times \overline{H}(S_c^-) \tag{5}$$

where $\overline{H}(S_c)$ is the magnetic field just inside the surface S_c, and $\overline{n}$ is the unit outward normal vector to the surface of the cylinder.

The electric current defined by (5) produces zero electric and magnetic fields outside the cylinder and produces the original electric and magnetic fields inside the cylinder. Enforcing the condition that the scattered electric field is zero just outside the cylinder an EFIE can be developed:

$$\overline{n} \times \overline{E}^s(\overline{J}) = 0 \text{ on } S_c^+ \tag{6}$$

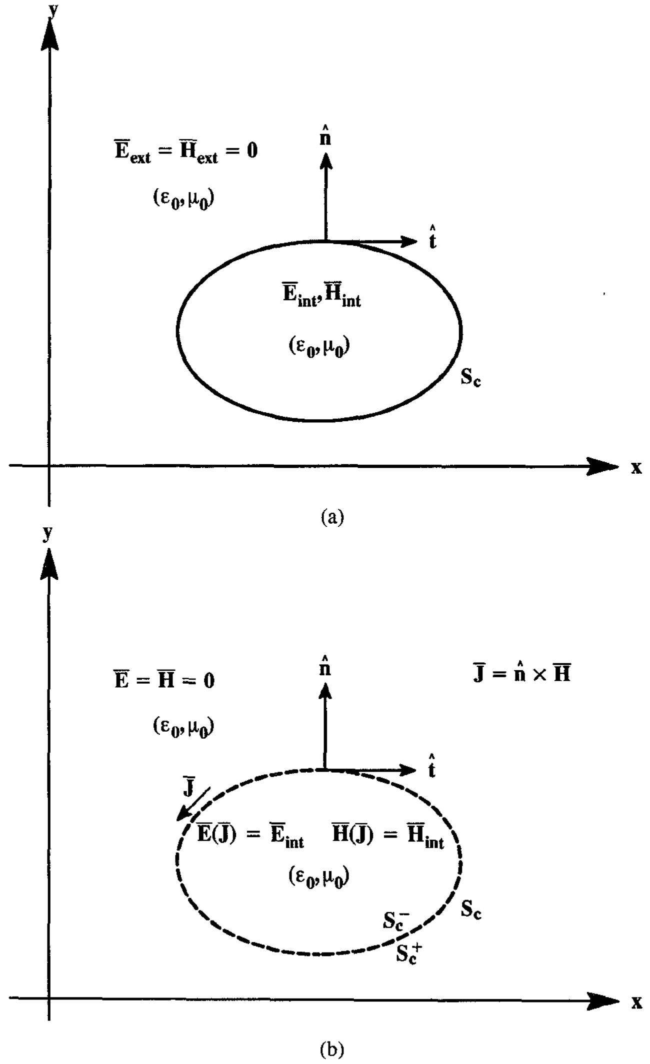

Fig. 2. (a) The original problem. (b) The equivalent problem.

where the explicit expression for the scattered electric field is the following,

$$\bar{E}^s[\bar{J}(\bar{r})] = \frac{1}{j\omega\epsilon_o}(k_o^2 + \bar{\nabla}\bar{\nabla}) \iint_{S_c} \bar{J}(\bar{r}')g(\bar{r}, \bar{r}')\, ds'. \tag{7}$$

Here $k_o = \omega\sqrt{\mu_o\epsilon_o}$ is the free space wave number and $g(r, r') = \exp(-jk_o|r - r'|)/(4\pi|r - r'|)$ is the three-dimensional free space Green's function.

Since TM_z and TE_z modes can propagate in the waveguide, $\boldsymbol{E}$ represents the axial electric field for TM_z modes and represents the transverse electric field for TE_z modes. In the following two sections, EFIE defined by (6) will be rewritten and solved for both (TM_z and TE_z) propagating modes.

IV. TM Formulation

Let a TM_z mode propagate in the waveguide. For this case the equivalent surface current has only a z component. Assuming an $e^{-j\beta z}$ behavior for waves along the z-direction, the equivalent electric current has a z-dependence,

$$J_z^e(x, y, z) = J_z(x, y)e^{-j\beta z}. \tag{8}$$

Substituting (8) into (7) and taking only the z component of the scattered field into account the integral equation (6) can be rewritten in the following form,

$$E_z^s(x, y, z) = \frac{1}{j\omega\epsilon_o}\left(k_o^2 + \frac{\partial^2}{\partial z^2}\right)A_z(x, y, z) = 0 \text{ on } S_c^+ \tag{9}$$

where the magnetic vector potential A_z is given by

$$A_z(x, y, z) = \oint_C J_z(x', y') \int_{-\infty}^{\infty} e^{-j\beta z'} \frac{e^{-jk_o R_{3d}}}{4\pi R_{3d}}\, dz'\, dl' \tag{10}$$

Here R_{3d} is the distance between the source and field points,

$$R_{3d} = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}.$$

It is impotant to note that it is enough to take the z component of the scattered electric field into account for the integral (6) because the x and y components of the electric and magnetic fields can be obtained from E_z [10, ch. 9].

In (10) the value of the infinite integral can be given in a closed form because,

$$\int_{-\infty}^{\infty} e^{-j\beta z'} \frac{e^{-jk_o R_{3d}}}{4\pi R_{3d}}\, dz' = \frac{e^{-j\beta z}}{2\pi} K_o[R_{2d}\sqrt{\beta^2 - k_o^2}]. \tag{11}$$

Here K_o is the zeroth order modified Bessel function of the second kind: [11, p: 107], and R_{2d} is the distance between the source and field points in an arbitrary x-y plane ($R_{2d} = \sqrt{(x - x')^2 + (y - y')^2}$). Equation (11) can be treated as the Fourier-transform of the three-dimensional Green's function for the z variable [12].

Since the propagation constant β is a real number (air-filling is assumed), the modified Bessel function K_o can be expressed by the zeroth order Hankel function of the second kind $H_o^{(2)}$ [13, p: 375],

$$K_o[jR_{2d}\sqrt{k_o^2 - \beta^2}] = \frac{\pi}{2j} H_o^{(2)}[R_{2d}\sqrt{k_o^2 - \beta^2}]$$

$$\text{if } -\frac{\pi}{2} < \arg(\sqrt{k_o^2 - \beta^2}) \le \pi. \tag{12}$$

Substituting (12) and (11) into (10) the expression for the magnetic vector potential is given by

$$A_z(x, y, z) = \frac{e^{-j\beta z}}{4j} \oint_C J_z(x', y') H_o^{(2)}[R_{2d}\sqrt{k_o^2 - \beta^2}]\, dl'. \tag{13}$$

Suppressing the term $e^{-j\beta z}$ in (13) and substituting it into (9) the original integral equation can be rewritten into the

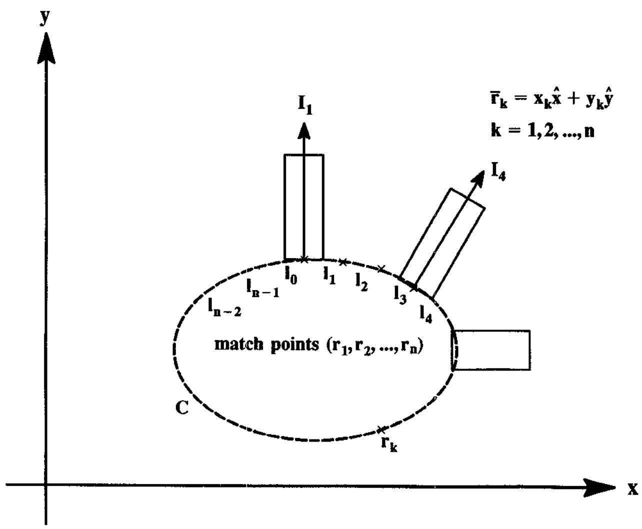

Fig. 3. Segmentation of C and expansion functions (TM case).

following form,

$$E_z^s(x, y) = -\frac{k_o^2 - \beta^2}{4\omega\epsilon_o} \oint_C J_z(x', y') H_o^{(2)} \cdot [R_{2d}\sqrt{k_o^2 - \beta^2}]\, dl' = 0 \text{ on } S_c^+. \quad (14)$$

Equation (14) is the integral equation pertaining to the TM_z mode for the hollow waveguide.

A method of moments procedure is used to transform the above integral equation into a matrix one. A pulse expansion for the current J_z along with a point matching testing procedure is chosen here. The contour C, making up the cross-section of the waveguide is replaced by a number of linear segments (n) as shown in Fig. 3 with the currents assumed to be constant on each segment [8]. Equation (14), after testing at the center of each linear segment, reduces to the form:

$$-\frac{(k_o^2 - \beta^2)}{4\omega\epsilon_o} \sum_{i=1}^{n} I_i \int_{l_{i-1}}^{l_i} H_o^{(2)}[\sqrt{(x_k - x')^2 + (y_k - y')^2} \cdot \sqrt{k_o^2 - \beta^2}]\, dl' = 0 \qquad k = 1, 2, \cdots n \quad (15)$$

In (15) the primed and unprimed variables represent the source and field points respectively, I_i represents the ith expansion coefficient for the current and (x_k, y_k) are the coordinates of the kth matching point. The rest of the quantities are as defined in Fig. 3.

At a fixed propagation constant β and angular frequency ω, (15) reduces to the matrix equation which can be solved for the expansion coefficients,

$$[\bar{Z}][\bar{I}] = [0] \quad (16)$$

where $\boldsymbol{Z}$ is the impedance matrix and $\boldsymbol{I}$ is the vector containing the expansion coefficients.

V. TE Formulation

Let a TE_z mode propagate in the waveguide. For this case the equivalent surface current has x, y and z components, while the scattered electric field has x and y components only. As in the previous section, assuming the $e^{-j\beta z}$ dependence for the wave along the z-direction and making a separation according to the transverse and axial directions, (7) can be rewritten into the following coupled form,

$$E_z^s = \frac{1}{j\omega\epsilon_o}\{-j\beta\bar{\nabla}_t\bar{A}_t + (k_o^2 - \beta^2)A_z\} \quad (17a)$$

$$\bar{E}_t^s = \frac{1}{j\omega\epsilon_o}\{(k_o^2 + \bar{\nabla}_t\bar{\nabla}_t)\bar{A}_t - j\beta\bar{\nabla}_t A_z\}. \quad (17b)$$

In the above equations A_z and $\boldsymbol{A}_t$ are the axial and transverse components of the magnetic vector potential and $\bar{\nabla}_t$ is the transverse del operator defined by (18),

$$\bar{\nabla}_t = \bar{x}\frac{\partial}{\partial x} + \bar{y}\frac{\partial}{\partial y}. \quad (18)$$

The explicit expressions for A_z and $\boldsymbol{A}_t$ are equations (10) and (19), respectively,

$$\bar{A}_t = \oint_C \bar{J}_t(x', y') \int_{-\infty}^{\infty} e^{-j\beta z'} \frac{e^{-jk_o R_{3d}}}{4\pi R_{3d}}\, dz'\, dl'. \quad (19)$$

Because for the TE_z propagation mode the electric field has no axial component, in (17a) E_z^s must be zero. Equating the right side of equation (17a) to zero the z component of the magnetic vector potential A_z can be expressed by the transverse component of the magnetic vector potential $\bar{A}_t$,

$$A_z = j\frac{\beta}{k_o^2 - \beta^2}\bar{\nabla}_t\bar{A}_t. \quad (20)$$

Substituting (20) into equation (17b) the transverse component of the electric field can be expressed only by the transverse component of the equivalent current via the magnitude vector potential $\boldsymbol{A}_1$,

$$\bar{E}_t^s = -j\omega\mu_o\left[\bar{A}_t + \frac{1}{(k_o^2 - \beta^2)}\bar{\nabla}_t(\bar{\nabla}_t\bar{A}_t)\right]. \quad (21)$$

Following the same idea as in the previous section, the infinite integral in equation (19) can be given in a closed form,

$$\bar{A}_t = \frac{e^{-j\beta z}}{4j}\oint_C \bar{J}_t(x', y') H_o^{(2)}[R_{2d}\sqrt{k_o^2 - \beta^2}]\, dl'. \quad (22)$$

Substituting (22) into (21), suppressing the term $e^{-j\beta z}$ and executing some straightforward manipulations (21) can be rewritten in the form,

$$\bar{E}_t^s = -\frac{\omega\mu_o}{4}\oint_C \bar{J}_t(x', y') H_o^{(2)}[R_{2d}\sqrt{k_o^2 - \beta^2}]\, dl' - \frac{\omega\mu_o}{4(k_o^2 - \beta^2)}\bar{\nabla}_t \oint_C \bar{\nabla}'\bar{J}_t(x', y') H_o^{(2)} \cdot [R_{2d}\sqrt{k_o^2 - \beta^2}]\, dl'. \quad (23)$$

Enforcing the boundary condition,

$$\bar{n} \times \bar{E}_l^s(\bar{J}_l) = 0 \text{ on } S_c^+ \tag{24}$$

the following EFIE is obtained for TE_z mode propagation,

$$\bar{t}(x, y)E_l^s[\bar{J}_l(x, y)] = 0 \text{ on } C. \tag{25}$$

In (25) $t(x, y)$ is the unit tangent vector defined by (4) and E_l^s is the transverse component of the scattered electric field given by (23).

As in the previous section a method of moments procedure is used to transform the integral (25) into a matrix one. A pulse expansion for the current J_l along with a point matching testing procedure is chosen. The contour C, making up the cross-section of the waveguide is replaced by a number of linear segments (n) as shown in Fig. 3. It is important to note that an approximation is made while choosing the expansion functions for the divergence of the electric current in (23). Since the divergence of a pulse function are two delta functions, the term containing the divergence of the current should be expanded in terms of delta functions. Instead of representing it in this form an approximation is made such that the divergence of the electric current is expanded in terms of pulse doublets in such a way that the moment of the pulse doublets is equal to the magnitude of the delta function [8]. Choosing a set of delta functions for weighting functions, (23) reduces to the numerical form,

$$\begin{aligned} -\frac{\omega\mu_o}{4} \sum_{i=1}^{n} I_i \int_{l_{i-1}}^{l_i} \bar{l}_k \bar{l}_i' H_o^{(2)}[\sqrt{(x_k - x')^2 + (y_k - y')^2} \\ \cdot \sqrt{k_o^2 - \beta^2}]\, dl' - \frac{\omega\mu_o}{4(k_o^2 - \beta^2)} \sum_{i=1}^{n} I_i \int_{l_i - 1/2}^{l_i + 1/2} \Gamma_i(l') \\ \cdot \{H_o^{(2)}[\sqrt{x_{k+1/2} - x')^2 + (y_{k+1/2} - y')^2}] \\ \cdot \sqrt{k_o^2 - \beta^2} - H_o^{(2)} \\ \cdot [\sqrt{(x_{k+1/2} - x')^2 + (y_{k+1/2} - y)^2} \\ \cdot \sqrt{k_o^2 - \beta^2}]\}\, dl' = 0. \end{aligned} \tag{26}$$

In (26) a finite difference approximation for the transverse del operator is used. Here l_k and the l_i' represent the unit directional vectors for the kth and ith elements respectively, I_i represents the expansion coefficient for the current and (x_k, y_k) are the coordinates of the kth matching point. The rest of the quantities are as defined in Fig. 4.

At a fixed propagation constant β and angular frequency ω, (26) reduces to the matrix equation (16), which can be solved for the expansion coefficients.

VI. Electromagnetic Fields Distribution

The first step in the calculation of the attenuation constant (α) is to obtain a relation between the propagation constant (β) and angular frequency (ω). The wavenumber k_o appears as an argument in the matrix equations (15) and

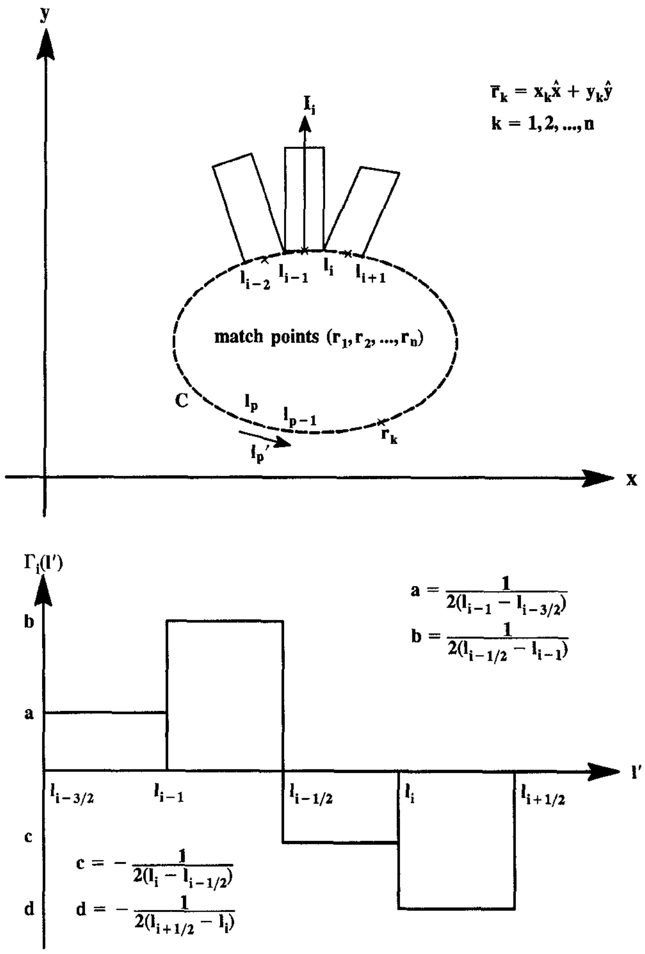

Fig. 4. Segmentation of C and expansion functions (TE case).

(26). For a non-trivial solution to exist for the vector I, the moment matrix Z has to be singular [1]. Hence

$$\det[\bar{\bar{Z}}] = 0. \tag{27}$$

A numerically robust way of finding the value of k_o at which the determinant becomes zero is to find the smallest eigenvalue. For this computational purpose the matrix equation (15) and (26) can be rewritten into the following forms,

$$\bar{\bar{Z}}\bar{I} = \lambda_{\min}\bar{I} \tag{28}$$

where $\lambda_{\min}$ is the minimum eigenvalue of the matrix Z and I is the corresponding eigenvector. Assuming β is fixed, then the angular frequency ω at which $\lambda_{\min}$ is the smallest one gives a relation between β and ω. Since the range for β is known ($0 \leq \beta \leq k_o$), the relation can easily be found. Because the moment matrix Z is unsymmetric and complex and the eigenvalues are also complex, the absolute value of the minimum eigenvalue is used in the algorithm for finding the relation between β and ω.

The Muller's method is used to find the angular frequency ω at which $\lambda_{\min}$ goes to zero [7]. This method is an iterative technique which converges quadratically in the vicinity of a root, does not require the evaluation of any derivatives and searches for complex roots even when these roots are not simple. A detailed explanation about the algorithm for obtaining the relation between β and ω using Muller's method can be found in [1], [7].

Once the β-ω relation is known, the next step in the computation of α is the calculation of the electromagnetic fields inside and on the surface of the waveguide. This is necessary to compute P_L and P_T.

In (28) $\boldsymbol{I}$ represents the eigenvector corresponding to the minimum eigenvalue $\lambda_{\min}$. Hence $\boldsymbol{I}$ represents the equivalent current coefficients producing the exact fields inside and on the surface of the waveguide. This vector can therefore be used to calculate the fields.

TM Fields

The various components of the electric and magnetic fields produced by a TM_z wave propagating in the waveguide can be obtained from the eigenvector $\boldsymbol{I}$ via the Maxwell's equation,

$$\overline{E} = \frac{1}{j\omega\epsilon_o}(\overline{\nabla} \times \overline{H}) \tag{29}$$

and via the equation which defines the magnetic vector potential ($\boldsymbol{A}$),

$$\overline{H} = \overline{\nabla} \times \overline{A}. \tag{30}$$

Since a TM_z wave propagates in the waveguide, the magnetic field has no longitudinal (z) component. After expanding the equivalent electric current (J_z) by pulse basis functions and making some simplifications, the various components of the electric and magnetic fields become,

$$H_z = 0 \tag{31a}$$

$$E_z = -\frac{(k_o^2 - \beta^2)}{4\omega\epsilon_o} \sum_{i=1}^{n} I_i \int_{l_{i-1}}^{l_i} H_o^{(2)}[R_{2d}\sqrt{k_o^2 - \beta^2}]\, dl' \tag{31b}$$

$$\overline{H}_l = \frac{\sqrt{(k_o^2 - \beta^2)}}{4j} \sum_{i=1}^{n} I_i \int_{l_{i-1}}^{l_i} \frac{\overline{y}(x - x') - \overline{x}(y - y')}{R} \cdot H_1^{(2)}[R_{2d}\sqrt{k_o^2 - \beta^2}]\, dl' \tag{31c}$$

$$\overline{E}_l = -\frac{\beta}{\omega\epsilon_o}\overline{z} \times \overline{H}_l. \tag{31d}$$

In the above equations R_{2d} represents the distance between the source and field points, $\boldsymbol{x}$ and $\boldsymbol{y}$ represent the x- and y-directed unit vector, respectively and $H_1^{(2)}$ is the first order Hankel function of the second kind. Here I_i is the ith component of the eigenvector $\boldsymbol{I}$.

The tangential component of the magnetic field on the surface of the waveguide is given by the electric currents,

$$\overline{H}_{\tan} = -\overline{n} \times \overline{J} \tag{32}$$

where $\boldsymbol{J}$ is given by the eigenvector and $\overline{n}$ is the unit outward normal vector to the surface of the waveguide.

TE Fields

The electromagnetic field produced inside the hollow waveguide for a TE_z mode propagating in the waveguide can be calculated from the eigenvector $\boldsymbol{I}$ via Maxwell's equation,

$$\overline{H} = -\frac{1}{j\omega\mu_o}(\overline{\nabla} \times \overline{E}) \tag{33}$$

and (30).

Since a TE_z wave propagates in the waveguide, the electric field has zero longitudinal component. After expanding the equivalent electric current ($\boldsymbol{J}_l$) by pulse basis functions and executing some simple and straightforward manipulations the various components of the electric and magnetic fields become

$$E_z = 0 \tag{34a}$$

$$\overline{E}_l = \frac{\omega\mu_o}{4} \sum_{i=1}^{n} I_i \int_{l_{i-1}}^{l_i} \overline{l}_i' H_o^{(2)}[R_{2d}\sqrt{k_o^2 - \beta^2}]\, dl' - \frac{\omega\mu_o}{4\sqrt{k_o^2 - \beta^2}} \sum_{i=1}^{n} I_i \int_{l_{i-1/2}}^{l_{i+1/2}} \Gamma_i(l') \cdot \frac{\overline{x}(x - x') + \overline{y}(y - y')}{R} H_l^{(2)}[R_{2d}\sqrt{k_o^2 - \beta^2}]\, dl' \tag{34b}$$

$$H_z = \frac{\sqrt{k_o^2 - \beta^2}}{4j} \sum_{i=1}^{n} I_i \int_{l_{i-1}}^{l_i} \left(\frac{\overline{R}}{R}\overline{n}'_i\right) H_l^{(2)} \cdot [R_{2d}\sqrt{k_o^2 - \beta^2}]\, dl' \tag{34c}$$

$$\overline{H}_l = \frac{\beta}{\omega\mu_o}\overline{z} \times \overline{E}_l. \tag{34d}$$

In the above equations l_i' is the tangential unit vector on the ith subsection, and $\overline{n}_i'$ is the unit outward normal on the ith subsection. The tangential magnetic field on the surface of the waveguide is given by (32).

VII. Numerical Results

After determining the various components of the electric and magnetic fields produced by either TM_z or TE_z wave propagating in the waveguide, the attenuation constant can be calculated using (1).

Since the zeroth order Hankel function of the second kind ($H_o^{(2)}$) has been used to determine the various components of the electromagnetic fields, they turn out to be complex quantities. However, as is well known, the fields inside a hollow waveguide are real quantities. Hence the electric and magnetic fields inside and on the surface of the waveguide have been normalized with respect to the largest one (in magnitude) existing at any point. This ensures that the electromagnetic fields produced by the wave propagating in the waveguide are no longer complex.

The attenuation constant which is given by (1) is not affected by this normalization. Equation (12) can be re-

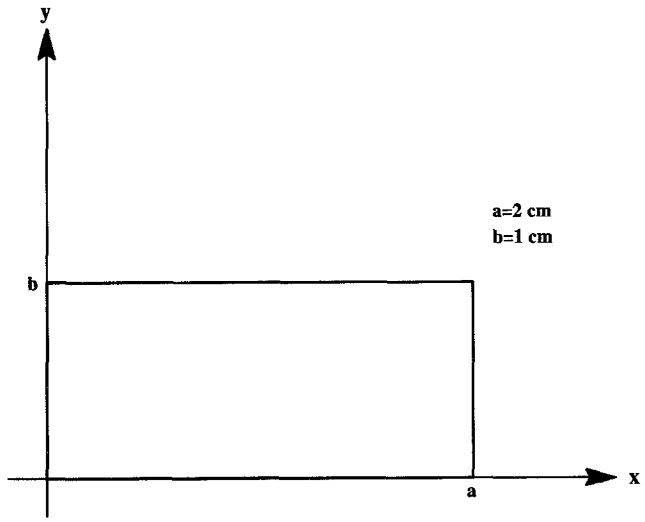

Fig. 5. Rectangular waveguide.

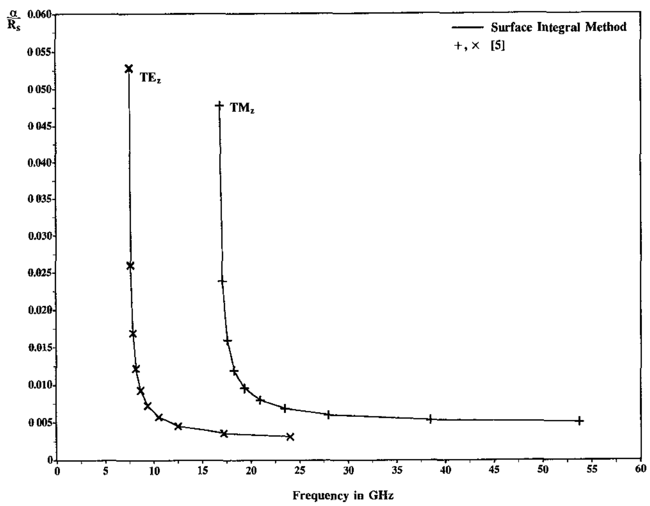

Fig. 6. Normalized attenuation constant for the rectangular waveguide as a function of frequency.

written in the form,

$$\alpha = \frac{P_{\text{ln}}}{2P_{\text{tn}}} \tag{35}$$

where P_{ln} and P_{tn} are the normalized power lost per unit length and normalized power transmitted, respectively. The explicit expressions for them are the following,

$$P_{\text{ln}} = \tfrac{1}{2}R_s \oint_C |H_{\tan,n}|^2 \, dl \tag{36a}$$

$$P_{\text{tn}}\tfrac{1}{2} \iint_S \text{Re}\,(\overline{E}_n \times \overline{H}_n^*)\bar{z}\, ds. \tag{36b}$$

In the above equations $H_{\tan,n}$ is the normalized magnetic field tangential to the waveguide walls assuming that the walls are perfectly conducting, $\boldsymbol{E}_n$ is the normalized electric field inside the waveguide and $\boldsymbol{H}_n^*$ is the complex conjugate of the normalized magnetic field existing inside the waveguide.

A mesh was generated and superimposed on the waveguide. The electric and magnetic fields were computed at discrete points inside and on the surface of the waveguide. The field distributions were then integrated to compute P_{ln} and P_{tn}. The unit for α in all the results provided is in nepers/unit length.

The first result pertains to a rectangular hollow waveguide with dimensions $a = 2$ cm and $b = 1$ cm (Fig. 5). The results obtained for α using the power-loss method along with a surface integral formulation have been compared with analytical results given in [5] for the waveguide shown in Fig. 5. The attenuation constant computed for the first TM$_z$ and first TE$_z$ mode propagating in the waveguide are shown in Fig. 6. The attenuation constant α has been normalized with respect to the surface resistance R_s to yield a more useful and more general diagram. In Fig. 6, this normalized attenuation constant α / R_s can be seen as a function of frequency. The cutoff frequency f_c for the TM$_z$ mode is 16.77 GHz and for the TE$_z$ mode

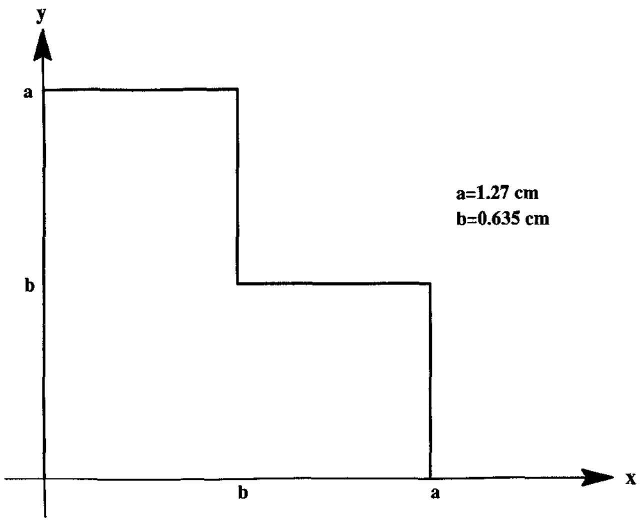

Fig. 7. L-shaped waveguide.

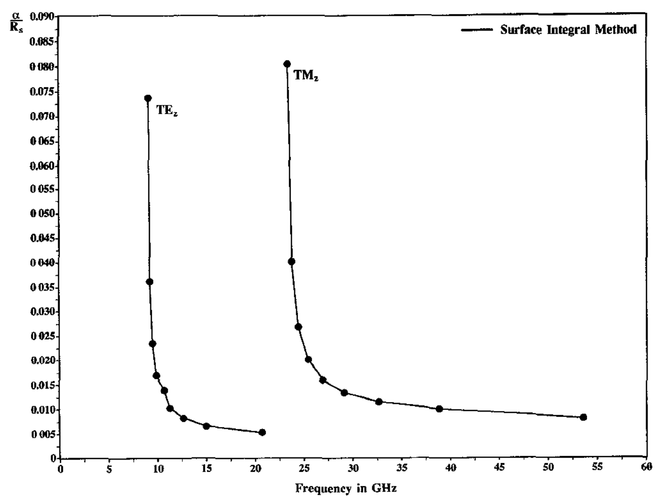

Fig. 8. Normalized attenuation constant for the L-shaped waveguide as a function of frequency.

is 7.5 GHz. The results obtained using the surface integral method compare very well with the analytical results.

The next example pertains to an L-shaped hollow waveguide. The precise geometry with the dimensions are given in Fig. 7. For this geometry there is no analytical results available. The normalized attenuation constant for the first TM_z mode as a function of frequency are shown in Fig. 8. The cutoff frequency f_c for the TM_z mode is 23.328 GHz and for the TE_z mode is 9.033 GHz. The shape of the curves in Fig. 8 are the same as in Fig. 6, which means a similar behavior of the attenuation constant for the L-shaped waveguide as for the rectangular waveguide.

VIII. Summary and Conclusion

The power-loss method coupled with the surface integral formulation has been used to compute the attenuation constant in hollow waveguides of arbitrary cross-section. A simple point-matching testing procedure with pulse expansion functions has been chosen to transform the integral equation into a matrix one. A fast iterative technique has been developed to obtain the relation between the propagation constant and frequency. Both TE and TM cases have been considered. The numerical results show agreement with available analytical results.

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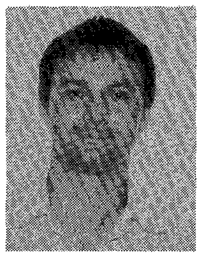
Tapan K. Sarkar (S'69–M'76–SM'81–F'92), photograph and biography not available at the time of publication.



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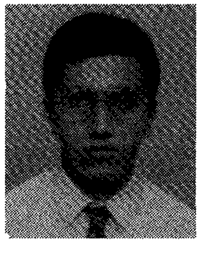

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